



**GCE AS/A level**

0983/01



S16-0983-01

**MATHEMATICS – S1**  
**Statistics**

A.M. WEDNESDAY, 15 June 2016

1 hour 30 minutes

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The events  $A$  and  $B$  are such that

$$P(A) = 0.3, P(B) = 0.4.$$

Evaluate  $P(A \cup B)$  in each of the following cases.

- (a)  $A$  and  $B$  are mutually exclusive. [2]
- (b)  $A$  and  $B$  are independent. [3]
- (c)  $P(A|B) = 0.25$ . [4]
2. In a certain population, 45% are male and 55% are female. It is known that 3% of the males have red hair while 5% of the females have red hair. One of the members of the population is selected at random.
- (a) Calculate the probability that the selected person has red hair. [3]
- (b) Given that the selected person has red hair, calculate the probability that this person is female. [3]
3. The random variable  $X$  has a Poisson distribution with mean 2. The random variable  $Y = aX + b$ , where  $a, b$  are positive constants.
- (a) Given that the mean and the variance of  $Y$  are both equal to 8, determine the values of  $a$  and  $b$ . [6]
- (b) Bethan states that, because the mean and variance of  $Y$  are equal, it must follow a Poisson distribution. Explain briefly why this is not the case. [1]
4. The committee of a social club contains 8 members, of which 4 are Welsh, 2 are English and 2 are Irish. A sub-committee of 3 members is to be set up and it is decided to select the 3 members at random.
- (a) Calculate the probability that the sub-committee contains
- (i) no Welsh members,
- (ii) one member of each nationality. [5]
- (b) Jack is a member of the committee. Find the probability that he is selected for the sub-committee. [2]

5. Customers arrive at a shop such that the number of arrivals in a time interval of duration  $t$  minutes follows a Poisson distribution with mean  $0.2t$ .

(a) Without the use of tables, determine the probability that the number of arrivals between 10:00 a.m. and 10:30 a.m. is

(i) exactly 5,

(ii) more than 3. [6]

(b) Given that the probability of exactly 5 arrivals in an interval of duration  $t$  minutes is 0.0602, where  $t < 30$ , use tables to determine the value of  $t$ . [3]

6. In a shooting range at a country fair, customers pay £5 to fire 8 shots at a target. Let  $X$  denote the number of shots which hit the target. Prizes are awarded according to the following rules.

If  $X < 2$ , no prize is awarded.

If  $X = 2$ , a prize of £10 is awarded.

If  $X > 2$ , a prize of £25 is awarded.

Jim decides to spend £5 to fire 8 shots. You may assume that the probability of one of his shots hitting the target is 0.12 and that successive shots are independent.

(a) Calculate the probability that he wins

(i) no prize,

(ii) a £10 prize,

(iii) a £25 prize. [5]

(b) Calculate his expected profit, giving your answer correct to two decimal places. [2]

7. The discrete random variable  $X$  has the following probability distribution.

$x$	1	2	3	4	5
$P(X = x)$	0.3	0.2	0.1	$a$	$b$

where  $a, b$  are positive constants.

(a) (i) Show that

$$a + b = 0.4.$$

(ii) Given that  $E(X) = 2.85$ , obtain a second equation involving  $a$  and  $b$ . Hence determine the value of  $a$  and the value of  $b$ . [5]

(b) Given that  $X_1$  and  $X_2$  are independent observations of the random variable  $X$ , determine  $P(X_1 + X_2 \leq 4)$ . [4]

**TURN OVER**

8. Jane is solving a problem in which she has to calculate  $P(X = 2)$  where  $X$  has a Poisson distribution with mean 3. Unfortunately, she has no statistical tables with her and her simple calculator has no  $e^x$  button and it can only carry out arithmetic operations. She decides to use an appropriate binomial distribution to give an approximate value for  $P(X = 2)$ . She takes  $n = 50$ .
- (a) What value of  $p$  should she take? [2]
- (b) Write down and evaluate an arithmetic expression giving her approximate value correct to four decimal places. [2]
- (c) Show that the approximation is within 1% of the value obtained from the appropriate Poisson table. [3]
9. The time,  $X$  hours, in the evening that Bill spends on his homework has probability density function  $f$  given by

$$f(x) = k(2x - 1) \quad \text{for } 1 \leq x \leq 2,$$

$$f(x) = 0 \quad \text{otherwise,}$$

where  $k$  is a constant.

- (a) (i) Find an expression in terms of  $k$  and  $x$  for  $F(x)$ , valid for  $1 \leq x \leq 2$ , where  $F$  denotes the cumulative distribution function of  $X$ .
- (ii) Hence show that  $k = \frac{1}{2}$ . [5]
- (b) Determine
- (i)  $E(X)$ ,
- (ii) the median of  $X$ ,
- (iii) the probability that, on a randomly chosen evening, Bill spends longer than 1.5 hours on his homework. [9]

**END OF PAPER**